Finite Hilbert Stability.

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Abstract. The classical construction of the moduli space of stable curves via Geometric Invariant Theory relies on the asymptotic stability result of Gieseker that the m-th Hilbert Point of a pluricanonically embedded curve is GIT-stable for all sufficiently large m. Several years ago, Hassett and Keel observed that if one could carry out the GIT construction with non-asymptotic linearizations, the resulting models could be used to run a log minimal model program for the space of stable curves. A fundamental obstacle to carrying out this program is the absence of a non-asymptotic analogue of Gieseker's stability result, i.e. how can one prove stability of the m-th Hilbert point for small values of m?

In this talk, we'll begin with a basic discussion of geometric invariant theory as well as how it applies to construct the moduli space of smooth curves in order to introduce and motivate the essential stability question in which this procedure rests on. The main result of the talk is to show: the m-th Hilbert point of a general smooth canonically or bicanonically embedded curve of any genus is GIT-semistable for all $m > 1$. We will then proceed to give a short proof of this result. This talk is based on joint work with Maksym Fedorchuk and David Smyth.