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Graphical Structure in Polynomial Systems: Chordal Networks

Abstract: We introduce a novel representation of structured polynomial ideals, which we refer to as chordal networks. The sparsity structure of a polynomial system is often described by a graph that captures the interactions among the variables. Chordal networks provide a computationally convenient decomposition of a polynomial ideal into simpler (triangular) polynomial sets, while preserving its underlying graphical structure. We show that many interesting families of polynomial ideals admit compact chordal network representations (of size linear in the number of variables), even though the number of components could be exponentially large. Chordal networks can be computed for arbitrary polynomial systems. Furthermore, they can be effectively used to obtain several properties of the variety, such as dimension, cardinality, components, and radical ideal membership. We apply our methods to examples from algebraic statistics and vector addition systems; for these instances, algorithms based on chordal networks outperform existing techniques by orders of magnitude.