Hoy: (1) Completar MaxFlow – MinCut (CORREGIR Error!)
(2) Problemas de momentos (Teorema Tchakaloff)

(1) $\beta^* = \max \sum_{i:(s,i)\in E} f_{si}$ s.a. $\underline{-C_{ij} \leq f_{ij} \leq C_{ij}}$

$\forall v \in V\setminus\{s,t\} \left(\sum_{i:(i,v)\in E} f_{iv} - \sum_{vj\in E} f_{vj} = 0\right)$

$\Leftrightarrow$

dual ✓ (ver clase anterior)

$\forall (v_i : i \in V\setminus\{s,t\})$

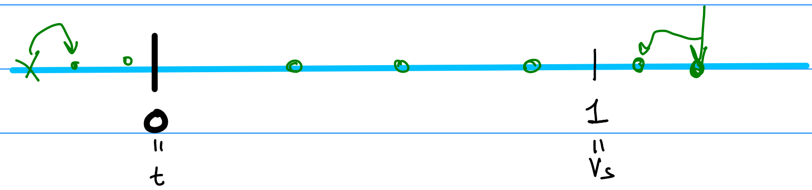
$$\alpha^* = \min_{V\in\mathbb{R}^{V-2}} \left\{ \sum_{\substack{ij\in E\\ \{i,j\}\cap\{s,t\}=\emptyset}} C_{ij}\,|v_j - v_i| + \sum C_{sj}\,|v_t - 1| + \sum_{j:(s,j)\in E} C_{jt}\,|v_j| \right\}$$

min global de función convexa

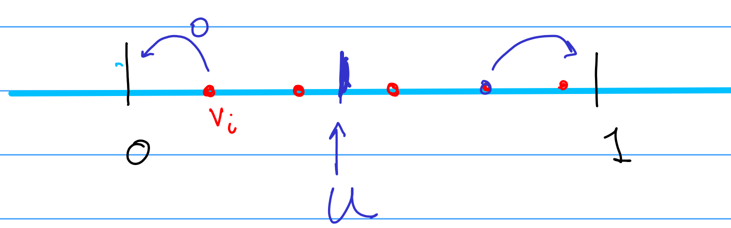
Por dualidad fuerte $\alpha^* = \beta^*$ $\forall i$

Af: Existe un óptimo v^* con $v_i \in \{0,1\}$.
($\{i : v_i = 1\} \cup \{s\} \sqcup \{i : v_j = 0\} \cup \{t\}$ es un corte con capacidad mínima)

Dem (error en clase anterior!)
Sea $\vec{v}$ un minimizador de $\mathcal{F}$ con valor α^*



Sin pérdida de generalidad asumo $v \in [0,1]$ $\forall v \in V\setminus\{s,t\}$.

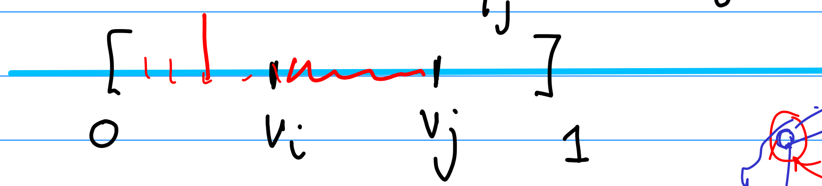


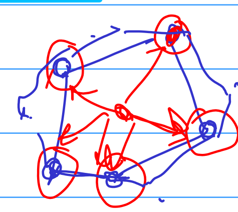
$$\vec{V}(u)_j = \begin{cases} 1, & \text{si } u \leq v_j \\ 0, & \text{si } u > v_j \end{cases}$$

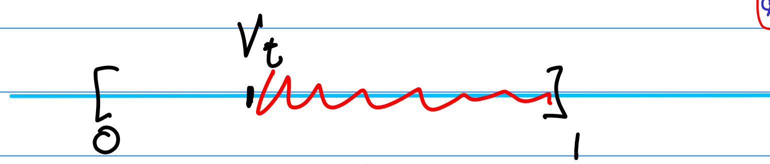
$$\mathbb{E}\left[\underbrace{\Psi(\vec{V}(u))}_{\text{fnc. obj del problema}}\right] \stackrel{?}{=}$$

$$= \sum_{ij} c_{ij}\, \underbrace{\mathbb{E}\left[|V_j(u) - V_i(u)|\right]}$$

$$\sum_{ij} c_{ij} |v_i - v_j| = \Psi(v) = \alpha^*$$





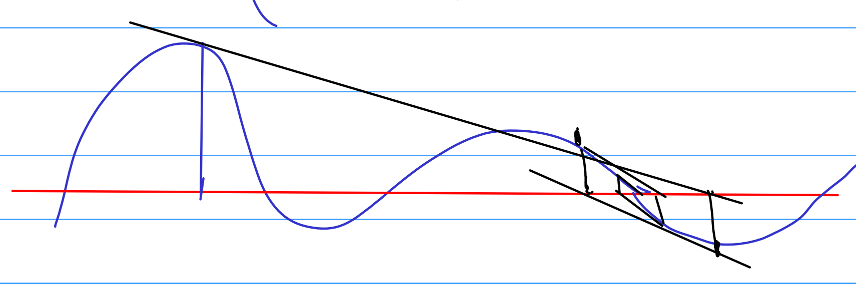


$$\mathbb{E}\left[|V_t(u) - 1|\right] = 1 - v_t = |v_t - 1|$$

$$\forall \omega \in \Omega \left(\Psi(\underbrace{V(u(\omega))}_{\in \{0,1\}^V}) \geq \alpha^* \right)$$

$$\mathbb{E}\left[\Psi(V(u(\omega)))\right] = \alpha^*$$

$$\forall \omega \in \Omega \left(\Psi(V(u(\omega))) = \alpha^* \right)$$



(2) Problemas de momentos

Ejemplo. Sea X una variable aleatoria en $[0,1]$ con $\left(\mathbb{E}[X]=0,\ \mathbb{E}[X^2]=1.\right)$ ← momentos

Qué puede decir sobre $\mathbb{P}\{X\geq \alpha\} \leq \frac{\mathbb{E}[X^2]}{\alpha^2} = \frac{1}{\alpha^2}$

Ejemplo. Sea $K\subseteq \mathbb{R}^n$ compacto

$$\max_{\mu} \int \underbrace{f}_{\text{conocida}}\, d\mu = L_\mu(f) \quad \text{s.a.} \quad \int \underset{L_\mu(1)=1}{1}\, d\mu = 1$$

$$\int g_j\, d\mu = c_j,\ j=1,\dots,T \qquad \left(\int g_j\, d\mu = L_\mu(g_j)\right)$$

$f, g_1,\dots,g_T \in C(K,\mathbb{R}) = C(K)$.

Problema de momentos

Hechos: Medidas de Radon $= [C(K)]^*$

$$\mu \rightsquigarrow \left(f \longmapsto \int f\, d\mu\right) = L_\mu$$

Por Riesz-Markov-Kakutani

Medidas de Radon positivas $= \left[\left\{ L\in C(K)^* : L(f)\geq 0\ \forall f\in C(K) \text{ con } f(x)\geq 0 \text{ en } K\right\}\right]$

$\langle h_1,\dots,h_K\rangle = V$ $\qquad \mathcal{M}(K)$

Sea $V \subseteq C(K)$ un subespacio de dimensión finita que contenga alguna constante no nula

$$V^* \supseteq C(K)^* \qquad \overbrace{\left(L_\mu(h_1),\dots,L_\mu(h_K)\right)}^{\text{momentos de } \mu}$$

$$L_\mu|_{V^*} \longleftarrow L_\mu$$

← momentos $\qquad \cup$

$$\mathcal{M}_V(K) \longleftarrow \mathcal{M}(K)$$

$$\mu|_{V^*} \longleftarrow L_\mu$$

Restricción MUY importante porque permite escribir problemas de momentos

Hemos construido:

$$\underbrace{V^*}_{\substack{\uparrow \\ \text{dim finita}}} \supseteq \underbrace{\mathcal{M}_V(K)}_{\uparrow}$$

cono en dim finita
"Momentos de medidas positivas respecto a $h_1, \dots, h_n$"

$$\left[\begin{array}{l} \max \quad L_y(f) \quad : \quad \left.\begin{array}{l} L_y(1) = 1 \\ L_M(g_i) = c_i \\ i = 1 \dots T \end{array}\right] \\ \\ L_y \in \mathcal{M}_{\langle 1, g_1 \dots, g_T, f \rangle}(K) \end{array}\right.$$

tiene dim finita

Cómo es este CONO???