

Teorema: Sea R un anillo local noetheriano con $\dim(R) = 1$
 R es un DVR $\Leftrightarrow R$ es normal (int. cerrado en $k = \mathcal{Q}(R)$)

Dem: " \Rightarrow " R DVR \Rightarrow todo ideal $I \subseteq R$ es $I = (x^k)$
 donde x es cualquier generador de $\mathfrak{m} \Rightarrow R$ es un DIP
 $\Rightarrow R$ es DFU $\Rightarrow R$ es normal.

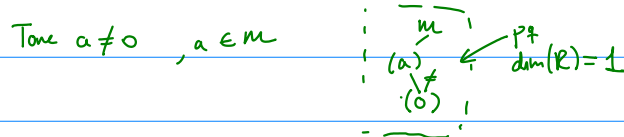
" \Leftarrow " Lema: R normal $\Rightarrow \exists x \in R$ $\mathfrak{m} = (x)$

Idea: Cómo encontrar el generador del maximal?

tone $a \neq 0, a \in \mathfrak{m}$ $(a) = (x^k)$ para algún k

$\mathfrak{m}^{k-1}(x^{k-1}) \not\subseteq (a)$ $b \in (\mathfrak{m}^{k-1}) \setminus (a)$

$b = x^{k-1}u$ $\boxed{x^k = \frac{a}{b}} \in R = \frac{x^k u'}{x^{k-1}u} = x \frac{u'}{u}$
 $(x^k) = \mathfrak{m}$ \nearrow generador de ideal



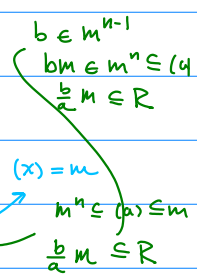
$\text{rad}(a) = \mathfrak{m}$, como R noeth

$\exists n: \mathfrak{m}^n \subseteq (a) \subseteq \mathfrak{m}$

Sea $n = \min \{n \in \mathbb{N}: \mathfrak{m}^n \subseteq (a)\}$

$\mathfrak{m}^{n-1} \not\subseteq (a)$

$\exists b \in \mathfrak{m}^{n-1}, b \notin (a)$



$\boxed{x := \frac{a}{b}} \in k$ Aj: $x \in R$ y $(x) = \mathfrak{m}$

Mostramos que $\frac{1}{x}(\mathfrak{m}) = R$

Dem: Si $\boxed{\frac{1}{x}\mathfrak{m} \neq R}$ $\frac{1}{x}\mathfrak{m} \subseteq \mathfrak{m}$

Si $\frac{1}{x}\mathfrak{m} \subseteq \mathfrak{m} \Rightarrow \frac{1}{x}$ es entero sobre R

[y por lo tanto $\frac{1}{x} \in R$] $\leftarrow R$ integralmente cerrado.

pero $\frac{1}{x} = \frac{b}{a}$ luego $b = (\frac{b}{a})a$ \boxtimes
 $b \in (a)$

$\boxed{\frac{1}{x}\mathfrak{m} = R} \Rightarrow \boxed{\mathfrak{m} = (x)}$

Teorema (Lasker-Hauptideal)

Sea M un A -mod f.g. y sea $\Phi: M \rightarrow M$
 un hom de A -mods. Entonces \exists submódulo

$\Phi^n + a_1 \Phi^{n-1} + \dots + a_{n-1} \Phi + a_n I_A = 0$

Aplicación: \mathfrak{m} es un R -mod f.g.

$\Phi: \mathfrak{m} \rightarrow \mathfrak{m}$
 $a \mapsto \frac{1}{x} \cdot a$

Así que

$$\bar{\Phi}^n + a_1 \bar{\Phi}^{n-1} + \dots + a_n \bar{\Phi} + a_0 = 0$$

$$\left[\left(\frac{1}{x}\right)^n + a_1 \left(\frac{1}{x}\right)^{n-1} + \dots + a_n \left(\frac{1}{x}\right) + a_0 \right] \cdot p = 0 \quad \forall p \in \mathfrak{m}$$

$$\left(\frac{1}{x}\right)^n + a_1 \left(\frac{1}{x}\right)^{n-1} + \dots + a_n \left(\frac{1}{x}\right) + a_0 = 0 \in \mathfrak{m}$$

$$\Rightarrow \frac{1}{x} \in \mathfrak{R}$$

\mathfrak{R} local, 1-dim, noeth y $\mathfrak{m} = (x)$

Af: $\forall z \in \mathfrak{R} \left(\exists u \in U(\mathfrak{R}), k \in \mathbb{N} : z = ux^k \right)$

$$z \in \mathfrak{R} \rightarrow z \in U(\mathfrak{R}) \checkmark$$

$$\rightarrow z \in \mathfrak{m}$$

$$z = z_2 \cdot x$$

$$z_2 \in U(\mathfrak{R}) \checkmark \quad z = xu$$

$$z_2 \notin U(\mathfrak{R})$$

$$z_2 \in \mathfrak{m} \quad z_2 = z_3 x$$

$$z = z_3 x^2$$

$$\rightarrow$$

$$\rightarrow \dots$$

$$(z) \subseteq (z_2) \subseteq (z_3) \subseteq \dots$$

Lemma $\boxed{z = ux^k}$

Los ideales de \mathfrak{R} son (x^k)

$$I = (z_1, \dots, z_n) = (u_1 x^{k_1}, \dots, u_n x^{k_n}) = (x^s)$$

$$\nu: \left. \begin{array}{l} \mathcal{Q}(\mathfrak{R}) \longrightarrow \mathfrak{K} \\ \nu\left(\frac{p}{q}\right) \longmapsto a-b \\ p \in \mathfrak{R} \quad p = ux^a \\ q \in \mathfrak{R}, q \neq 0 \quad q = u'x^b \\ \frac{p}{q} = x^{a-b} u'' \end{array} \right\} \mathfrak{R} \text{ es un DVR } \checkmark$$